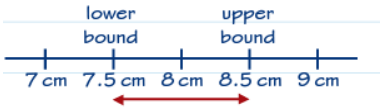
Unit 7 – Area and Volume

Error intervals (U657, U587)

Upper and lower bounds are a measure of accuracy. E.g. the width (w) of a postcard is given as 8cm to the nearest cm



$7.5 \leq w < 8.5$

	+	−	×	÷
Overall upper bound	UB + UB	UB − LB	UB × UB	UB ÷ LB
Overall lower bound	LB + LB	LB − UB	LB × LB	LB ÷ UB

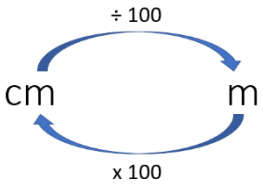
Overall lower bound of $a + b$ = lower bound of a + lower bound of b.

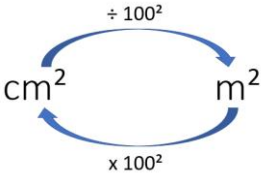
E.g. a roll of ribbon is 150cm long correct to 2 significant figures. A 21 cm piece of ribbon is cut off the role, correct to the nearest cm. Calculate the lower bound for the amount of ribbon remaining on the roll.

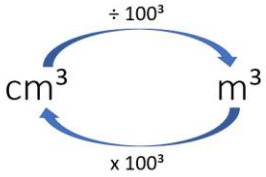
Converting Units (U388, U248, U468, U663)

- When converting from a larger unit to a smaller unit (e.g. m^2 to cm^2), you multiply.
- When converting from a smaller unit to a larger unit (e.g. mm^2 to cm^2), you divide

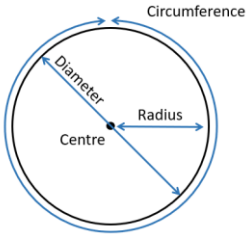
10mm = 1cm
100cm = 1m
1000m = 1km
1000g = 1kg
1000ml = 1 litre

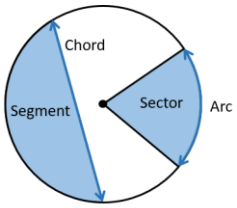




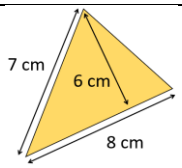
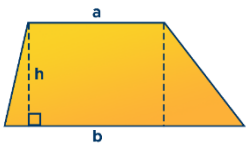
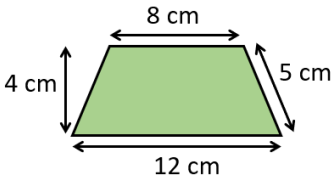
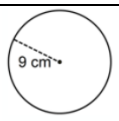
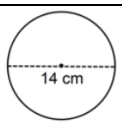
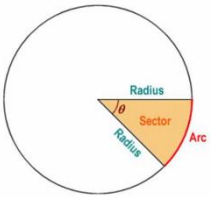
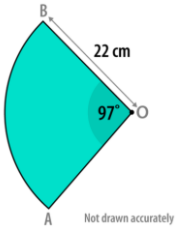


Parts of a circle



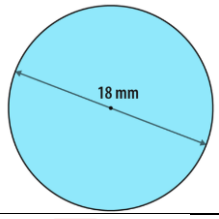


Radius	Length from centre to edge	**Arc**	Part of the circumference
Diameter	'Width', through centre	**Sector**	Area between two radii & an arc
Circumference	Perimeter	**Chord**	Line between two points on the circumference
		Segment	Area between a chord & an arc

Formulas	Example
Area of a triangle (U945, U575) $\frac{1}{2}bh$	
Area of a trapezium (U265, U904) $\frac{1}{2}(a+b)h$ 	
Area of a circle (U950) Diameter is twice the length of the radius Area of a circle = πr^2	 
Area of a sector (U373) $\frac{angle}{360}$ x πr^2 	

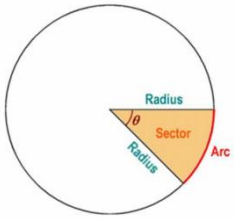
Circumference of a circle (U604)

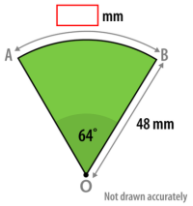
Circumference = πd



Arc length (U221)

$\frac{angle}{360}$ x πd

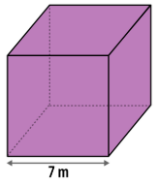




Cube/cuboid (U786, U929)

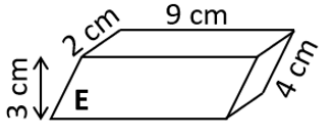
Volume = Length x width x height

Surface area = find the area of each face then add all the areas together



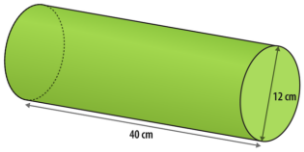
Volume of a prism (U174, U259, U142)

Cross section x length



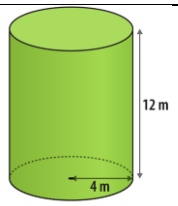
Volume of a cylinder (U915)

Cross section x length
= πr^2 x length



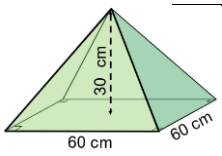
Surface area of a cylinder (U464)

$2\pi r^2 + \pi dh$



Volume of a pyramid (U484, U871)

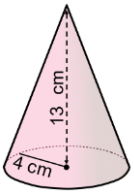
$\frac{1}{3}$ x base area x vertical height



Volume of a cone (U116)

$\frac{1}{3}$ x base area x vertical height

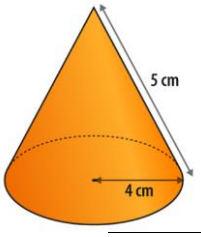
$\frac{1}{3}$ x πr^2 x vertical height



Surface area of a cone (U523)

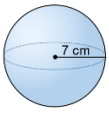
Curved surface area (given) = πrl
Area of a circle = πr^2

Total surface area = $\pi rl + \pi r^2$



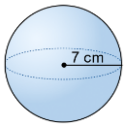
Volume of a sphere (given) (U617, U426)

$\frac{4}{3}\pi r^3$



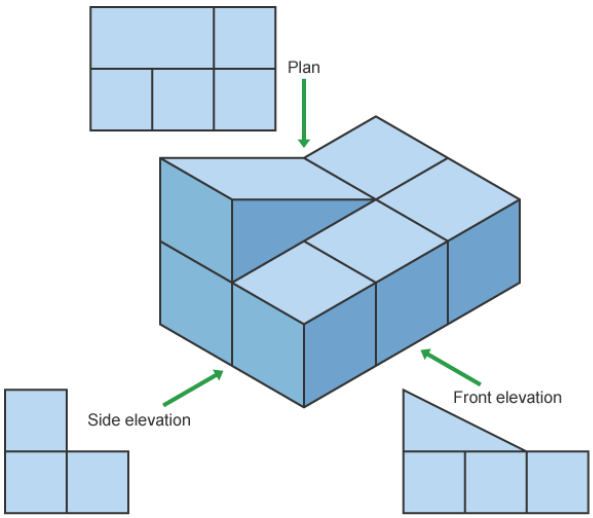
Surface area of a sphere (given) (U893, U771)

$4\pi r^2$



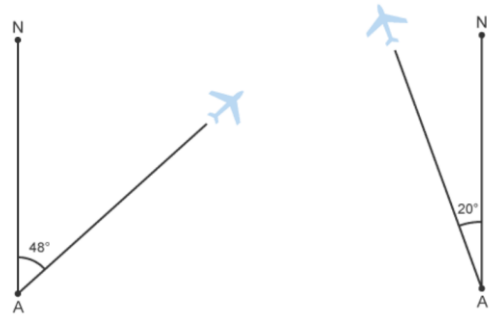
Unit 8 – Transformations

Plans and elevations (U743)

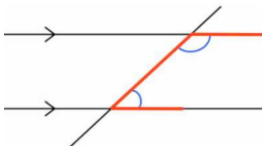


Bearings (U107, U525, U164)

- Measure from the north line
- Measure clockwise
- Always has three digits



Other knowledge you might need for bearings:



Co-interior angles sum to 180 degrees

Translation (U196)

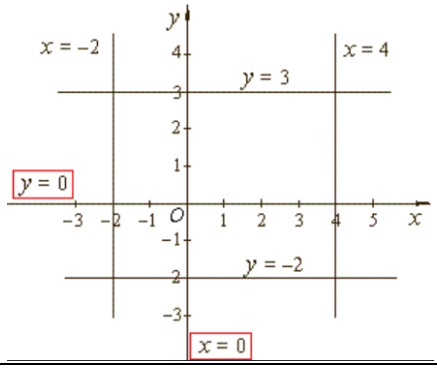
When describing, include:

- Translate
- Column Vector $\begin{pmatrix} x \\ y \end{pmatrix}$

Reflection (U799)

When describing, include:

- Reflect
- Line of reflection



Rotation (U696)

When describing, include:

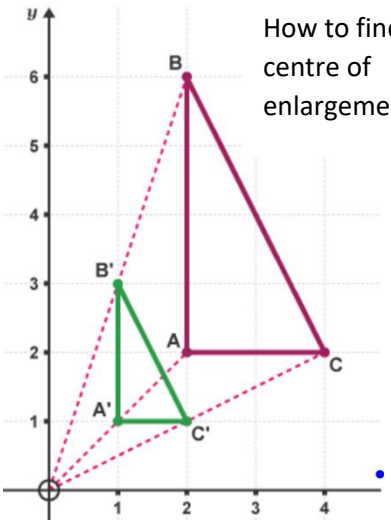
- Rotation
- Centre of rotation
- Anticlockwise or clockwise
- Degrees turned (90°, 180°, 270°)

Enlargement (U519, U134)

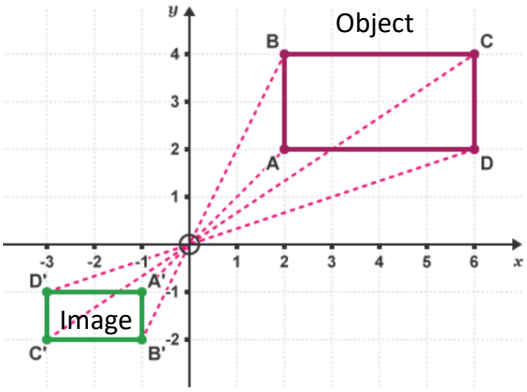
When describing, include:

Enlargement
Scale factor
Centre of enlargement

Enlargement by scale factor ½ from centre (0,0) →



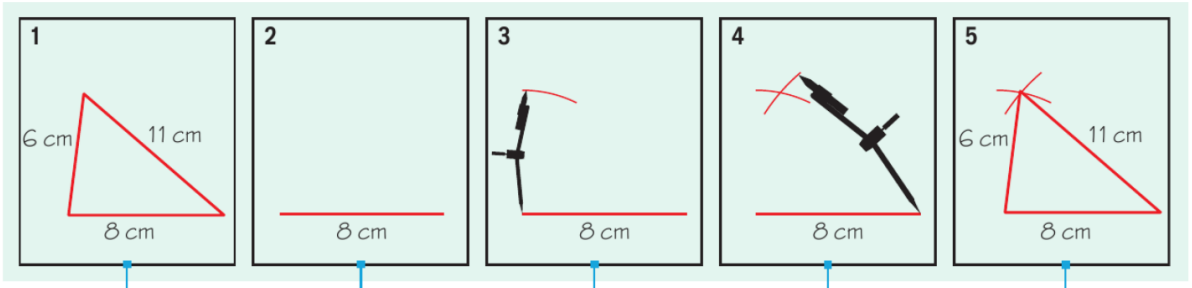
How to find the centre of enlargement:



The rectangle ABCD has been enlarged by a scale factor of $-\frac{1}{2}$.

Triangle construction (U187)

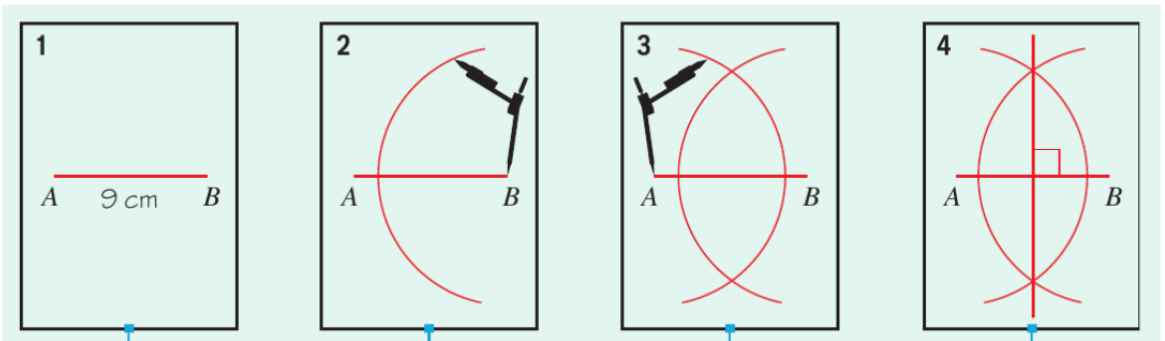
Construct a triangle with sides 11 cm, 8 cm and 6 cm.



1. Sketch the triangle.
2. Draw the 8 cm line.
3. Open your compasses to 6 cm. Place the point at one end of the 8 cm line. Draw an arc.
4. Open your compasses to 11 cm. Draw another arc from the other end of the 8 cm line.
5. Join the intersection of the arcs to each end of the 8 cm line. Don't rub out your construction arcs.

Perpendicular bisector (U245, U979)

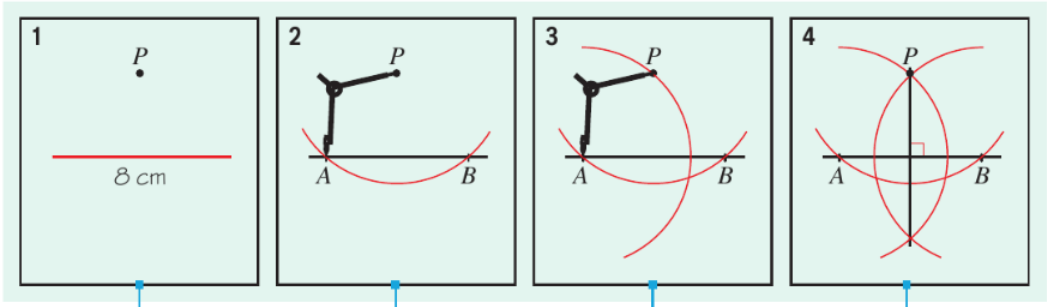
Draw a line AB 9 cm long. Construct its perpendicular bisector.



1. Use a ruler to draw the line.
2. Open your compasses to more than half the length of the line. Place the point at B and draw an arc above and below.
3. Keeping the compasses open to the same distance, move the point of the compasses to A and draw a similar arc.
4. Join the points where the arcs intersect. Don't rub out your construction arcs.

Perpendicular bisector between a point and a line

Draw a horizontal line 8 cm long and a point P that is 3 cm above the line. Then construct a line that is perpendicular to the horizontal line that passes through point P.



Use a ruler to draw an 8 cm line and then mark point P, 3 cm above this line.

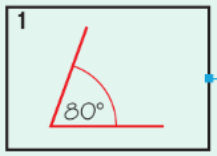
Open your compasses to a radius larger than the distance from P to the line (the larger the distance, the more accurate your diagram). Then, with your compasses on point P, draw an arc that cuts the line twice. Label the two intersection points A and B.

Put the compasses on point A and draw an arc above and below the horizontal line. Repeat, with compasses open the same distance and with the compass point at B.

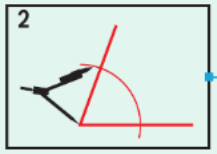
Join the intersection points of these two arcs. This line will go through P and will be perpendicular to the horizontal line.

Angle bisector (U787)

Draw an angle of 80°.
Construct the **angle bisector**.

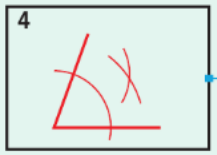


Draw an angle of 80° using a protractor.

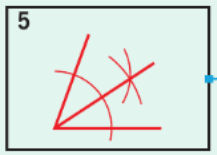


Open your compasses and place the point at the vertex of the angle. Draw an arc that crosses both arms of the angle.

Keep the compasses open to the same distance. Move them to one of the points where the arc crosses an arm. Make an arc in the middle of the angle.



Do the same for where the arc crosses the other arm.



Join the vertex of the angle to the point where the two small arcs intersect. This line is the angle bisector. Don't rub out your construction arcs.

Unit 9 – Equations and Inequalities

Solving linear inequalities (U759, U509)

Solve these inequalities and represent them on a number line:

$3 - 2x \leq 7$

When you multiply or divide by a negative number, reverse the inequality

$6 < 4x + 2 \leq 18$

Factorise quadratic expressions (U178, U963, U858)

Factorise $x^2 + 7x + 10$

We need to find the factors of 10, that add up to 7

$2 + 5 = 7$

So $(x + 2)(x + 5)$

10	
1	10
2	5
3	X
4	X
5	2

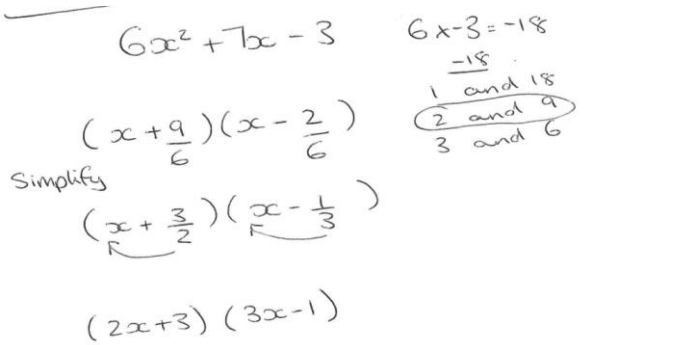
$x^2 + 3x - 10$

$x^2 - 49$

$4x^2 - 64$

Look out for difference of two squares! Are both terms square numbers?

(+)(-)



$6x^2 + 7x - 3$

$(x + \frac{9}{6})(x - \frac{2}{6})$

$(x + \frac{3}{2})(x - \frac{1}{3})$

$(2x + 3)(3x - 1)$

Completing the square (U397)

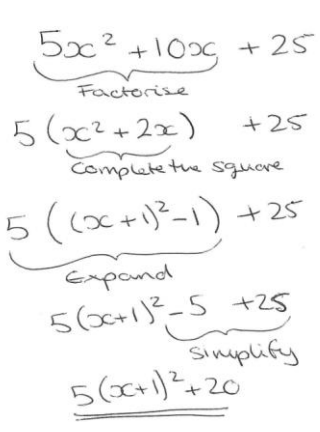
Write in completed square form:

$x^2 + 4x + 18$

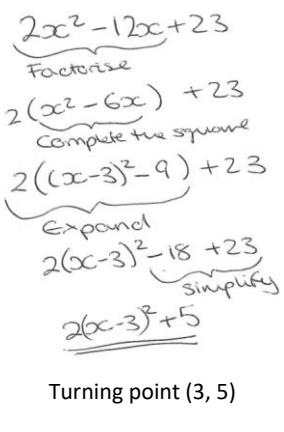
$(x + 2)^2 - 4 + 18$

$(x + 2)^2 + 14$

Turning point = (-2, 14)



Turning point (-1, 20)

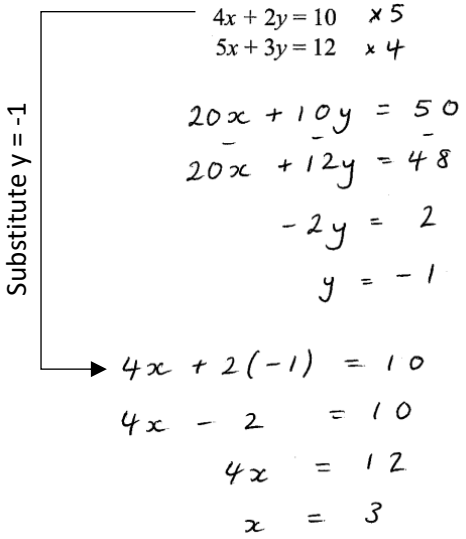


Turning point (3, 5)

Solve linear simultaneous equations (U760)

1. Write the equations one above the other and number them
2. (Multiply each term in the equation so the coefficients are the same)
3. **SAMES SIGNS SUBTRACT**

   DIFFERENT SIGNS ADD
4. Solve
5. Substitute into one of the equations and solve again



Solving quadratics - $ax^2 + bx + c = 0$

There are 4 methods to solving quadratics. Unless it is the perfect square, make sure the right-hand side of the equation equals 0.

1. Solve by factorising (U228, U960)

$x^2 + 4 = 4x$

1. Factorise
2. Write each bracket equal to 0 and then solve

2. Solve the perfect square

$(x + 2)^2 = 9$

1. Square root both sides – **remember ±**
2. Solve the +
3. Solve the -

3. Solve by completing the square (U589)

Solve $2x^2 + 10x = -5 - 2x$ by completing the square

Step 1: Make it equal to 0

$2x^2 + 10x = -5 - 2x$

$+5 \quad +5$

$2x^2 + 10x + 5 = -2x$

$+2x \quad +2x$

$2x^2 + 12x + 5 = 0$

Step 2: Complete the square

Factorise

$2(x^2 + 6x) + 5 = 0$ (complete square)

$2((x+3)^2 - 9) + 5 = 0$ (expand)

$2(x+3)^2 - 18 + 5 = 0$ (simplify)

$2(x+3)^2 - 13 = 0$

Step 3: Solve the perfect square

$2(x+3)^2 - 13 = 0$

$+13 \quad +13$

$2(x+3)^2 = 13$

$\div 2 \quad \div 2$

$(x+3)^2 = \frac{13}{2}$

$\sqrt{\quad} \quad \sqrt{\quad}$

$x + 3 = \pm\sqrt{\frac{13}{2}}$

$-3 \quad -3$

$x = \pm\sqrt{\frac{13}{2}} - 3$

4. Solve using the quadratic formula (U665)

Quadratic Formula =

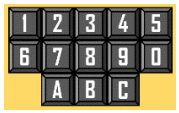
1. Compare with $ax^2 + bx + c$. Write the values of a, b, and c.
2. Substitute a, b and c into the quadratic formula
3. + gives one solution and – gives the other

Solve $x^2 + 4x + 2 = 0$. Give your solutions in surd form.

a = b = c =

Unit 10 – Probability

Product Rule (U369)



A safe has a keypad and a 4-character code.

a) How many different codes are there?

b) A code starts with a letter. How many different codes are there?

c) The code has no repeated characters. How many different codes are there?

Sample Space Diagrams (U104)

Sample space lists all the possible outcomes.

A dice and a coin are thrown at the same time.

a) Complete the Sample Space Diagram

Coin \ Dice	1	2	3	4	5	6
Head	H, 1					
Tail					T, 5	

b) P(Heads and a number greater than 2)=

c) P(NOT a 5 and NOT Heads)=

Mutually exclusive events (U683)

Two events are mutually exclusive if they cannot occur at the same time or simultaneously.
e.g.

You can **add** their probabilities.

A set of events is exhaustive when the events include every possible outcome. The probabilities of an exhaustive set of mutually exclusive events sum to 1 – because there is 100% chance one of them will happen.

The probability that the train arrives on time is $\frac{3}{10}$.
What is the probability it will be late?

Experimental and Theoretical Probability (U580,U166)

Theoretical probability is what is expected to happen.
Experimental probability/relative frequency is the result of an experiment.

George picks one marble at random out of a bag. He notes its colour and puts the marble back. He repeats this until he has picked a marble 80 times.

Colour	Frequency	Relative Frequency
Red	8	$\frac{8}{80} = \frac{1}{10}$
Blue	16	$\frac{16}{80} = \frac{1}{5}$
Green	44	$\frac{44}{80} = \frac{11}{20}$
Yellow	12	$\frac{12}{80} = \frac{3}{20}$
Total	80	

(a) Complete the relative frequency column of the table above.

(b) In the bag, there are 20 marbles in total. Estimate how many of each colour there are.

Red: $\frac{1}{10} \times 20 = 2$ Green: $\frac{11}{20} \times 20 = 11$

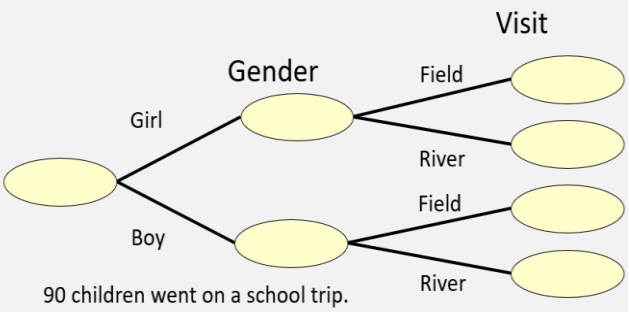
Blue: $\frac{1}{5} \times 20 = 4$ Yellow: $\frac{3}{20} \times 20 = 3$

Two-Way Tables (U981, U246)

	Stalls	Circle	Balcony	Total
Adults	36	39		112
Children	41		31	
Total		60		

What is the probability that a randomly chosen adult is sat on the balcony?

Frequency trees (U280)



90 children went on a school trip.
They either went to a field or a river.
There were 43 boys altogether.
20 girls and 21 boys went to visit the field.

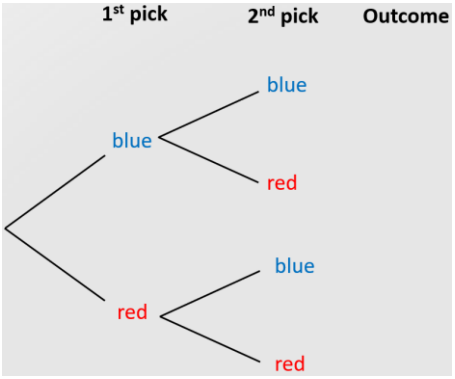
a) Complete the frequency tree

b) Use the diagram to find the probability that a child chosen at random would be a girl that visited the river

Probability trees for independent events (U558)

Independent events: when the probability of one event does not affect the probability of another event.
E.g. the probability that I eat a salad for dinner does not affect the probability that it rains.

A bag contains 3 blue counters and 4 red counters.
A counter is chosen at random, then replaced.
Another counter is then chosen.



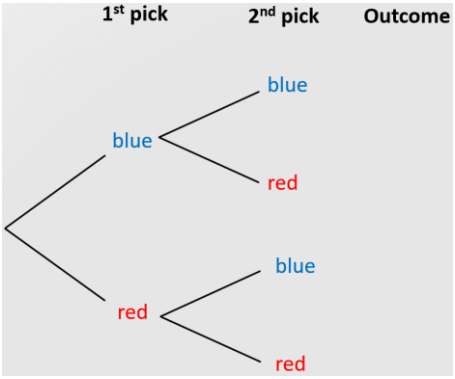
Find P(RR)

Find P(same colour)

Probability trees for dependent events (U729, U806)

Dependent events: when the probability of one event does affect the probability of another event.
E.g. If I took a blue marble from a bag, this would decrease the probability of selecting another blue marble again because there would be less blue marbles.

A bag contains 2 blue counters and 5 red counters.
Two counters are chosen at random.

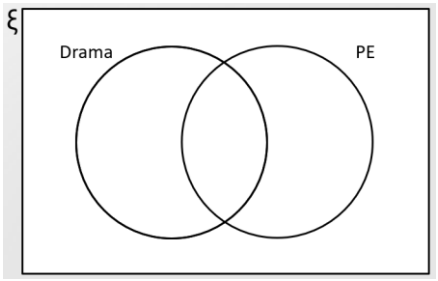


Find P(BR)

Find P(different colours)

Venn Diagrams (U476)

60 students were surveyed.
37 students study PE.
15 students study both Drama and PE.
17 students don't study Drama or PE.



P(studies Drama) =

Calculating probabilities (U510)

Mike rolls a die and flips a coin. What is the probability the dice lands on a 6 AND the coin shows tails?

Mike rolls a die once. What is the probability he gets a 2 OR a 3?

Venn Diagram Notation (U748)

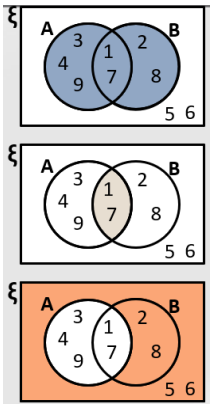
A set is a collection of 'things'.
ξ = universal set (contains all the elements)

- Does not contain duplicates
- The order of the elements does not matter

A 'member' or 'element' is an item in a set.

$\emptyset$ = empty set

$\in$ = is a member of
$\notin$ = is not a member of



P(A) – probability of A

n(A) – number of members in set A

Unit 11 - Multiplicative Reasoning

Percentages of amounts: calc (U349, U671)

To find a percentage, multiply by the decimal equivalent

Percentage increase = 100% + n% - then change to decimal
Percentage decrease = 100% - n% - then change to decimal

Appreciates = increases
Depreciates = decreases/reduces

Increase 800 by 3%

Increase 800 by 30%

Find 30% of 800

Decrease 800 by 3%

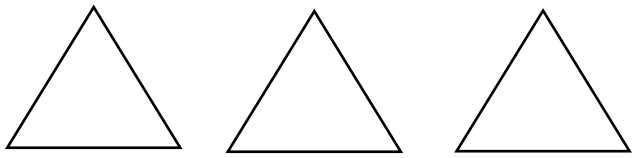
Decrease 800 by 30%

Repeated percentage changes

Future skills Tristan buys a flat for £135 000.
In the first year, the value of the flat appreciates by 12%.
In the second year, the value of the flat depreciates by 3%.
Work out the value of the flat after the 2 years.

Compound measures

Compound measures combine measures of two different quantities, e.g. speed is a measure of distance travelled and time



Compound interest (U332)

Matt invests £7500 in an account paying 2.8% **compound interest** per annum (yearly). How much does he have in the account after three years?

Compound measures (U151)

Convert 144,000 m/s to km/h

Maddy runs 3.5km in 15minutes.
What speed is she running in km/h?

Kinematics Formula

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

You don't need to learn these formulas but must know how to use them

a = acceleration (how fast velocity changes e.g. m/s²)
u = initial velocity (speed in a given direction at the start of motion)
v = final velocity (speed in a given direction at the end of motion e.g. m/s)
s = displacement, how far from the start point in a straight line e.g. m

A car starts from rest and accelerates at $5\,m/s^2$ for 200 m.
Work out the final velocity in m/s.

Problem-solving A bus travels with an acceleration of $2\,m/s^2$ and reaches a speed of 45 km/h in 5 seconds.
What was the initial velocity of the bus in m/s?

Combined measures

Papa Smurf walked 10m at 5 m/s from A to B, and then 5m at 10m/s from B to C.
What was Papa Smurf's average speed across the journey?

Create the table and fill in what you know

	A to B	B to C	Overall
Speed	5m/s	10 m/s	
Distance	10m	5m	
Time			

← Add the distances to get the total distances

← Add the times to get the total time

We need to find these two times
Time = Distance ÷ Speed

Do not add the speeds or work out the mean speed!

	A to B	B to C	Overall
Speed	5m/s	10 m/s	
Distance	10m	5m	15m
Time	2 seconds	0.5 seconds	2.5s

Average speed = $\frac{total\ distance}{total\ time}$

= 15 ÷ 2.5 = 6 m/s

Direct Proportion (U721, U407)

There is a direct proportion between two values when one is a multiple of the other

$t \propto r$ = t is directly proportional to r
K = constant of proportionality

1. Write an equation of proportionality.
2. Substitute $\boldsymbol{x}$ & $\boldsymbol{y}$ to find k.
3. Rewrite the equation using k and substitute $\boldsymbol{x}$ to find $\boldsymbol{y}$.

a is directly proportion to b. When $a = 8$, $b = 10$. Find the value of a when $b = 13$.

Indirect Proportion (U357, U138)

Inverse proportion – As one variable increases, the other decreases at a consistent rate

y is inversely proportional to x.
When $x = 6,\ y = 5$.
Find y when $x = 9$.

b is inversely proportional to the square root of a.
When $a = 4,\ b = 10$.
Find b when $a = 25$.

Unit 12 – Congruency and Similarity (U112)

Congruency (U790, U866)

Two shapes are congruent if they have the same shape and size.

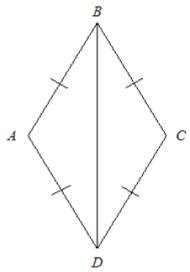
4 ways to prove congruency:

- Side, Side, Side (SSS) – all 3 sides are the same
- Side, Angle, Side (SAS) – two sides and the **included** angle are the same
- Angle, Side, Angle (ASA, AAS) – two angles and a corresponding side are the same
- Right angle, Hypotenuse, Side (RHS) – congruent if the triangles are right-angled and the hypotenuse and another side are the same

Proving Congruence (U887)

STEP 1: Choose your appropriate proof (e.g. SSS)
STEP 2: Justify each of three things.
STEP 3: Conclusion, stating the proof you used.

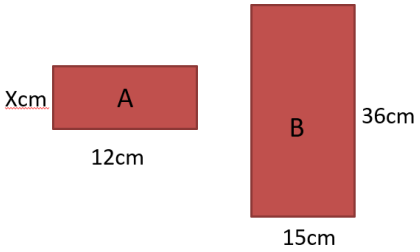
Prove triangle ABC is congruent to triangle BCD



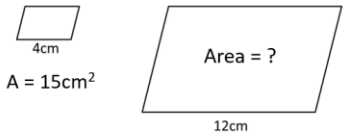
Similarity (U551, U578)

Shapes are similar if they are proportionally the same e.g. every side is twice as big.
The angles will always be exactly the same.

Rectangle A and B are similar
What is the missing side?



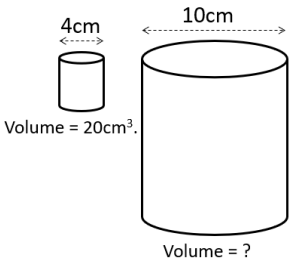
Similarity – Area Scale Factor (U110)



1. Length Scale Factor
2. Area Scale Factor
3. Area of the larger shape

Similarity – Volume Scale Factor (U110)

1. Length Scale Factor
2. Volume Scale Factor
3. Calculate the missing volume



Knowledge Retriever

DO NOT WRITE ON THIS PAGE SO YOU CAN REATTEMPT THE QUESTIONS AT A LATER DATE

Here are some of the example questions found in this knowledge organiser booklet, see if you can answer them without looking.

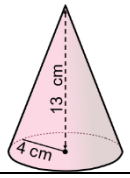
Unit 7:

The width (w) of a postcard is given as 8cm to the nearest cm. Write down the error interval for w.

What is the area of a sector with angle 97° and radius 22cm?

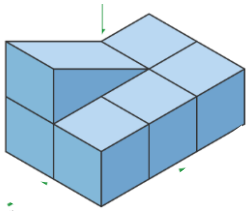
What is the arc length of a sector with angle 64° and radius 48mm?

What is the volume of this cone?

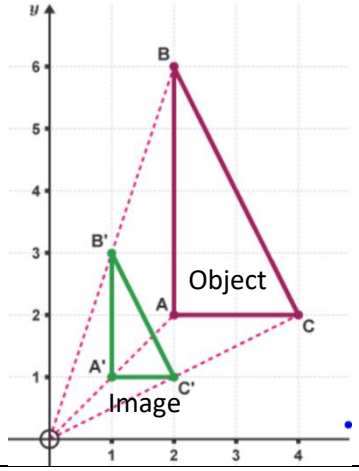


Unit 8:

Draw the plan for this 3D solid:



Describe this transformation:



Unit 9

Solve $6 < 4x + 2 \leq 18$

Factorise $x^2 + 3x - 10$
Factorise $6x^2 + 7x - 3$

Solve these simultaneous equations:
$4x + 2y = 10$
$5x + 3y = 12$

Write $x^2 + 4x + 18$

Solve by factorising $x^2 + 4 = 4x$

Solve $x^2 + 4x + 2 = 0$. Give your solutions in surd form.

Unit 10:

A bag contains 2 blue counters and 5 red counters. Henry chooses two counters randomly. Find the probability he chooses 2 counters which are different colours.

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37 students study PE.
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Find P(studies drama)

Unit 11

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Unit 12

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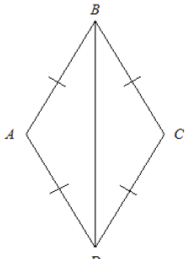

Calculate the missing volume